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Do models of vertical strategic interaction for national and store brands meet the market test?

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Abstract

This article develops a methodology for empirically examining some of the central assumptions commonly used in the theoretical literature on vertical strategic interaction. This methodology is used to test these assumptions by using data for six individual categories across 59 local markets in 1991 and 1992, focusing on the vertical and horizontal interaction between private label and national brands. There are three central findings. First, the vertical strategic interaction observed for national brands varies considerably across categories (a single form of interaction, "Vertical Nash," tends to be more common for private label brands). Second, we generally reject the use of proportional mark-up behavior by retailers. Third, we reject linear demands in a favor of a more flexible nonlinear form. These results suggest that models specifying proportional mark-up behavior and linear demands do not accurately reflect market reality. Further, because vertical strategic interaction between manufacturers and retailers seems to be idiosyncratic to the category and brand, future research should consider multiple forms of vertical interaction to produce reasonably general results.

1. Introduction

When setting shelf prices, do retailers simply apply nonstrategic pricing rules to manufacturer's wholesale prices? When manufacturers set their wholesale prices, do they anticipate retailer's reactions? Are the demand curves linear? Although the assumptions implicit

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in these queries are often helpful in making channel analyses tractable, the conclusions derived from any model must be questioned if the model's underlying postulates do not reflect actual market behavior.

This is particularly true in the economic literature of channels where most previous research has been theoretical in nature with little empirical work examining the assumptions on which the models are based. Without such investigations, might much of this analytic literature be based on a house of cards? We address this concern in this research by evaluating three central assumptions common to much of the analytic literature of channels:

> *Form of Vertical Strategic Interaction between Manufacturers and Retailers*. Modelers typically employ two different assumptions with regard to vertical strategic interaction (VSI), Manufacturer Stackelberg (MS), and Vertical Nash (VN). Under MS interaction, manufacturers are the price leaders within the channel and under VN interaction, manufacturers and retailers ignore each other's moves.[1] We present a methodology for testing these assumptions by using only retail price data for three different demand specifications.
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> *Use of Nonstrategic Proportional Mark-up Behavior by Retailers*. A common assumption used with regard to competitive interaction (e.g., Kadiyali, Vilcassim, and Chintagunta, 1999) is that retailers act nonstrategically by applying a fixed mark-up on manufacturer wholesale prices. Although this assumption makes it easier to assess horizontal competitive interaction, we do not know whether retailers consistently act in this nonstrategic fashion. Further, Sudhir (2000) demonstrated that there are identification problems inherent in the estimation of models that assume proportional mark-up pricing behavior by retailers. Thus, for each form of vertical channel interaction (MS or VN), we test for the use of a "rule of thumb" proportional mark-up pricing rule by retailers for private label and nationally branded products.
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> *Linear Demands*. The vast majority of economic channel models assume linear demand curves. Three such specifications can be found in (1) the Choi (1991) restricted linear demand model; (2) the Raju, Sethuraman and Dhar (1995) "Shubik" demand model of store brand introduction; and (3) a general, linear demand model. We examine how well such linear demand specifications characterize actual market behavior by comparing them to a general flexible non-linear functional demand form, the Linear Approximate Almost Ideal Demand Specification (LA/AIDS).

We begin by focusing on the Choi (1991) and Raju et al. (1995) linear demand models, deriving the associated MS and VN reaction functions for each under strategic profit maximizing behavior. We then demonstrate that the Choi (1991) and Raju et al. (1995) MS and VN models can be represented as special cases of a more general class of mark-up models. We also show that each can be represented as special cases of the general linear model (because of the complexity of the derivation of the general linear model, we present it and all the relevant derivations in the Appendix).

Empirical tests are then developed for MS and VN conduct and for the use of proportional mark-up rules by retailers within these games. With an eye to our empirical application, we focus on two brands, one a national brand and one a private label. This provides a direct connection between the theoretical framework used in both articles and the mark-up assumptions made in much of the empirical IO literature.

In the second portion of this article, we relax the above linear demand assumptions. Further, if we are to assess the applicability of the assumptions embodied in these models under a more realistic environment, we must also extend our analysis to consider more realistic forms of interaction (on the demand side in particular). Consequently, we present a comprehensive model of private label—national brand interaction by specifying a general class of flexible nonlinear demands, namely the LA/AIDS (see Cotterill et al., 2000). The reaction function associated with this non-linear model is sufficiently general to encompass the MS or VN behavior within the channel, thereby making it readily comparable to the Choi (1991) and Raju et al. (1995) models.

The empirical analysis is based upon IRI data for six individual categories (milk, butter, bread, pasta, margarine, and instant coffee) across 59 local geographic markets for 1991 and 1992. We estimate the demand and price reaction equations simultaneously for each category and for each of the models considered. In particular, a series of nested and nonnested tests are used to assess empirically the primary assumptions employed in the literature on vertical channel interaction and private label noted above.

From our results, we find that models specifying proportional mark-up behavior and linear demands do not accurately reflect market reality (at least in the six product categories studied here). Further, we show that VSI between manufacturers and retailers is idiosyncratic to the category and brand. By implication, future research should also consider multiple forms of vertical interaction.

2. Testing assumptions about vertical strategic interaction and demand functional form

Modeling the interaction between national brands and private labels is especially challenging because, unlike competition between two national brands, there is a vertical relationship between national brand manufacturers and retailers. As a result, one needs to be concerned about the vertical as well as the horizontal nature of competitive interaction. Choi (1991) analyzes linear demands under three different behavioral assumptions for the vertical pricing game. These models assume that manufacturers act as Stackelberg leaders within the channel (MS), retailers act as Stackelberg leaders within the channel (RS), and retailers and manufacturers play a VN game within the channel, respectively. Alternatively, Raju et al. (1995) use a restricted version of Shubik's linear-form demand model and specify MS conduct.

In the next sub-section, we derive the retail reaction functions for the Choi (1991) MS, Choi VN, and the Raju et al. (1995) MS models. For the sake of completeness, we also extend the Raju et al. model by deriving the VN version of their model. We further derive the Stackelberg and Nash reactions for the general linear model, but reserve this for the Appendix. When each these reaction functions are combined with the appropriate linear demand structure, we have a fully estimable demand and supply system that allows us to test for alternative demand forms and vertical channel behavior. These estimates become the baseline models to which we compare more complex representations of private label—national brand interactions.

2.1. The Choi (1991) MS and VN models

Choi (1991) assumes that the vertical game has three players. In our context, this implies a national brand manufacturer, a private label manufacturer, and a retailer that sells both the private label product and the national brand. Following in the tradition of McGuire and Staelin (1983) and Jeuland and Shugan (1988), Choi specifies linear demand functions that allows for product differentiation (we retain his notation here):

$$q_i = a - bp_i + \gamma p_j, \ \forall\, i, j = 1, 2, j \neq i \tag{1}$$

where q_i is demand for brand i at price p_i given that the price of the other brand j is p_j. Without loss of generality, we designate Brand 1 as the national brand (NB) and Brand 2 as the private label (PL). The retailer maximizes category profits, Π_R, by choosing values of p_1 and p_2 for given wholesales prices for the product, w_1, w_2:

$$\underset{p_1,\, p_2}{\text{MAX}}\ \Pi_R = (p_1 - w_1)q_1(p_1, p_2) + (p_2 - w_2)q_2(p_1, p_2) \tag{2}$$

The two manufacturers maximize their profits as follows (c_1 and c_2 denote the respective manufacturer marginal costs):

$$\underset{w_i}{\text{MAX}}\ \Pi_{mi} = (w_i - c_i)q_i(p_1, p_2) \qquad i = 1, 2 \tag{3}$$

The retailer's first order conditions (FOC) are:

$$-2bp_1 + 2\gamma p_2 + bw_1 - \gamma w_2 + a = 0 \tag{4}$$

$$-2bp_2 + 2\gamma p_1 + bw_2 - \gamma w_1 + a = 0 \tag{5}$$

The manufacturer's first order conditions (FOC) are:

$$\text{National Brand: } a - bp_1 + \gamma p_2 - bw_1 + bc_1 = 0 \tag{6}$$

$$\text{Private Label: } a - bp_2 + \gamma p_1 - bw_2 + bc_2 = 0$$

This approach produces six endogenous variables (q_1, q_2, p_1, p_2, w_1, and w_2) and six equations (two demand equations, two retailer FOC, and two manufacturer FOC). Assuming either MS conduct or VN conduct allows one to solve these six equations for reduced form equations that give the equilibrium values of the endogenous variables as functions of the exogenous variables. In principle, one can estimate the parameters of the model from these reduced form equations; alternatively, one can estimate the six "structural" equations, provided that the data are available.

Because data on wholesale prices are not generally available, we take a different tack and reduce the structural estimation problem to four equations involving the four available endogenous variables ($q_1 q_2$, p_1 p_2). Assuming MS conduct, we do this by using the same information from the retailer's price reaction functions for p_1, p_2 that one uses to solve for reduced form equations in the Stackelberg game. The retailer's price reaction for p_1 derived from the retailer's FOC (Equations 4 and 5) is:

$$p_1 = \frac{w_1}{2} + \frac{a}{2(b-\gamma)} \tag{7}$$

Note that this equation implies that the manufacturer actually determines the retail level price for his brand when w_1 is set. Thus, one can restate the manufacturer's profit maximization problem with p_1 as the manufacturer's choice variable by solving (7) for w_1 as a function of p_1:

$$w_1 = 2p_1 - \frac{a}{(b-\gamma)} \tag{8}$$

Substituting it into Equation 3 (for $i = 1$) one obtains:

$$\underset{p_1}{\text{MAX}}\ \Pi_{NB} = \left(2p_1 - \frac{a}{b-\gamma} - c_1\right) q_1(p_1 p_2) \tag{9}$$

The corresponding FOC is:

$$2a - 4bp_1 + 2\gamma p_2 + \frac{ab}{b-\gamma} + bc_1 = 0 \tag{10}$$

Solving this equation for p_1 gives the price reaction function between the manufacturer-controlled retail level brand price and the retail-controlled private label price. This reaction captures the manufacturer's best response for setting p_1 when retailers choose a value for p_2:

$$p_1 = \frac{\gamma}{2b} p_2 + \frac{a}{2b} + \frac{a}{4(b-\gamma)} + \frac{c_1}{4} \tag{11}$$

Similarly, the retailer's price reaction function for p_2 can be derived by solving the retailer's other first order condition for p_2. This gives the retailer's other vertical price reaction function:

$$p_2 = \frac{w_2}{2} + \frac{a}{2(b-\gamma)} \tag{12}$$

Solving for w_2 by substituting this into equation 3 (for $i = 2$), we obtain:

$$\underset{p_2}{\text{MAX}}\ \Pi_{PL} = \left(2p_2 - \frac{a}{b-\gamma} - c_2\right) q_2(p_1 p_2) \tag{13}$$

The retail-level price reaction function between the national brand and private label manufacturers is:

$$p_2 = \frac{\gamma}{2b} p_1 + \frac{a}{2b} + \frac{a}{4(b-\gamma)} + \frac{c_2}{4} \tag{14}$$

This provides us with an empirically estimable model that consists of the two demand equations (Equation 1, for i and j) and the two retail level brand price reaction functions (Equations 11 and 14). With appropriate cross equation restrictions, we can estimate the parameters, a, b, γ, in addition to the demand and price reaction elasticities. One can also use equation (8) to recover an estimate for w_1, the unobserved wholesale price for the national brand, and Equation (12) to recover an estimate for w_2, the unobserved wholesale price for the private label.

We also note that in the event that the private label manufacturer has no market power vis à vis the retailer, and sells its product at a price equal to marginal cost, then $w_2 = c_2$. The private label manufacturer's profit maximization problem vanishes and Equation (14) is replaced by Equation (12) with $w_2 = c_2$, giving the following price reaction curve:

$$p_2 = \frac{c_2}{2} + \frac{a}{2(b - \gamma)} \tag{15}$$

Thus, in this instance, the retail level price reaction functions are asymmetric—national brand prices react to changes in private label prices, while private label prices are exogenously determined.

If one assumes VN instead of MS conduct, then the reaction functions that correspond to (11) and (14) above are:

$$p_1 = \frac{\gamma}{3b} p_2 + \frac{a}{3b} + \frac{a}{3(b - \gamma)} + \frac{c_1}{3} \tag{11$'$}$$

$$p_2 = \frac{\gamma}{3b} p_1 + \frac{a}{3b} + \frac{a}{3(b - \gamma)} + \frac{c_2}{3} \tag{14$'$}$$

2.2. The Raju et al. (restricted Shubik) model

The only differences between the Choi (1991) and the Raju et al. (1995) models are that the former relaxes the identical intercept assumption in the demand system and explicitly assumes that private label manufacturers sells its product at a price equal to marginal cost.[2] This allows us to nest the Choi (1991) and Raju et al. (1995) models inside a general linear model, producing a convenient test of alternative views of demand structure and manufacturer-retailer interaction. In the case of the Raju et al. (1995) model, the appropriate structural model consists of two demand equations and two asymmetric retail price reaction functions for MS or VN conduct:

$$q_1 = a_1 + bp_1 + \gamma p_2 \tag{16}$$

$$q_2 = a_2 + bp_2 + \gamma p_1 \tag{17}$$

The MS reaction functions are specified as:

$$p_1 = \frac{\gamma}{2b} p_2 + \frac{a_1}{2b} + \frac{ba_1 + \gamma a_2}{4(b^2 - \gamma^2)} + \frac{c_1}{4} \tag{18}$$

$$p_2 = \frac{c_2}{2} + \frac{\gamma a_1 + ba_2}{b^2 - \gamma^2} \tag{19}$$

The VN reaction functions are specified as:

$$p_1 = \frac{\gamma}{3b} p_2 + \frac{a_1}{3b} + \frac{a_1 b + a_2 \gamma}{3(b^2 - \gamma^2)} + \frac{c_1}{3} \tag{18'}$$

$$p_2 = \frac{c_2}{2} + \frac{\gamma a_1 + b a_2}{2(b^2 - \gamma^2)} \tag{19'}$$

Note that the coefficient on private label price in the national brand price reaction equation is identical to the Choi (1991) coefficient for the MS and VN games.

2.3. The Choi (1991) and Raju et al (1995) models within a general class of mark-up models

Deriving the appropriate price reaction equations for each model (Choi, Raju et al., and general linear demands) we empirically test for the form of vertical interaction (MS and VN) between manufacturers and retailers within the channel. We can extend this to draw additional inferences about a common assumption used in theoretical models, namely that of the use of mark-up rules by retailers. This allows us to test whether the use of simple proportional mark-up pricing by retailers is consistent with the data in our empirical application. If retailers use a generalized mark-up for national brands, then:

$$p_1 = m_1 w_1 + m_0 \tag{20}$$

where m_1 and m_0 are proportional and fixed mark-up parameters respectively (see Kadiyali, Vilcassim, and Chintagunta 1999). Solving this price relation for w_1 gives:

$$w_1 = k_1 p_1 - k_0 \tag{21}$$

where $k_1 = (1/m_1)$ and $k_0 = (m_0/m_1)$. Substituting (21) into the manufacturer' profit maximization problem in (3), one can obtain the following retail level price reaction equation for the Choi (1991) demand system with MS conduct under generalized mark-up behavior by retailers:

$$p_1 = \frac{\gamma}{2b} p_2 + \left(\frac{a}{2b} + \frac{k_0}{a k_1}\right) + \frac{c_1}{2k_1} \tag{22}$$

Note that the reaction curve slope for the generalized mark-up model in (22) is identical to the slope ($\gamma/2b$) for the reaction curve under strategic profit maximization in (11). Since in our empirical work we employ instruments for marginal costs, we cannot identify k_1. However, we note that if:[3]

$$\frac{k_0}{k_1} = \frac{a^2}{4(b - \gamma)} = m_0, \text{ and } k_1 = 2, \tag{22'}$$

then generalized mark-up behavior is identical to profit maximizing behavior in a Choi (1991) MS game (since Equation 22 reduces to 11). Alternatively, under proportional mark-up pricing, $m_0 = 0$ hence $k_0 = 0$ and the reaction function reduces to:

$$p_1 = \frac{\gamma}{2b} p_2 + \frac{a}{2b} + \frac{c_1}{2k_1} \tag{23}$$

Thus, if the intercept in the national brand reaction function is equal to ($a/2b$), then the proportional mark-up model holds for the Choi (1991) specification. Note that if the private label manufacturers also play a Choi (1991) MS game, an analogous set of reaction functions exist.

Similarly, we can derive an analogous set of conditions in the event that the demand specification is consistent with MS behavior under restricted Shubik demands as in Raju et al. (1995). In this case, the reaction function for the national brand under the generalized mark-up model is:

$$p_1 = \frac{\gamma}{2b} p_2 + \frac{a_1}{2b} + \frac{k_0}{2k_1} + \frac{c_1}{2k_1} \tag{24}$$

For this to be consistent with profit maximizing behavior (i.e., identical to 18) one must have:

$$\frac{k_0}{k_1} = \frac{ba_1 + \gamma a_2}{2(b^2 - \gamma^2)}, \text{ and } k_1 = 2. \tag{25}$$

Now if we assume VN conduct between retailers and manufacturers in the Choi model the reaction function that corresponds to (22) is:

$$p_1 = \frac{m_1}{1 + m_1} \frac{\gamma}{b} p_2 + \frac{m_1}{1 + m_1} \frac{a}{b} + m_0 + m_1 c_1 \tag{23$'$}$$

where: m_1 the retailer's mark-up parameter. If $m_0 = 0$ we have the Choi proportional mark-up reaction equation. Analogous equations hold when p_2 is the dependent variable. For the Raju et al. VN model the reaction function that corresponds to (24) is:

$$p_1 = \frac{m_1}{1 + m_1} \frac{\gamma}{b} p_2 + \frac{m_1}{1 + m_1} \frac{a_1}{b} + m_0 + m_1 c_1 \tag{24$'$}$$

We use these relationships (and the corresponding ones derived in the Appendix for the more general linear model) in the next sub-section to develop tests for assessing (1) whether the demand structure observed is consistent with the Choi (1991), Raju et al. (1995) and/or a more general linear specification, (2) whether the vertical channel relationship is consistent with MS or VN behavior, and (3) whether retailers use proportional mark-up conduct within the MS/VN game. In the general linear model presented in the Appendix, we relax the identical intercept and own-price coefficient constraints.

2.4. Testing for demand structure, vertical conduct and nonstrategic mark-up behavior by retailers

The relationships detailed above provide for a series of convenient tests of demand structure and within channel behavior. To demonstrate, we first estimate the following linear demand and price reaction system:

$$q_1 = A_{10} - A_{11}p_1 + A_{12}p_2 + A_{13}D \tag{26}$$

$$q_2 = A_{20} - A_{21}p_2 + A_{22}p_1 + A_{23}D \tag{27}$$

$$p_1 = R_{10} + R_{11}p_2 + R_{12}c_1 + R_{13}D \tag{28}$$

$$p_2 = R_{20} + R_{21}p_1 + R_{22}c_2 + R_{23}D, \tag{29}$$

where all variables are defined earlier except for c_1 and c_2, which are instruments that assist in identifying the price equations and D, which denotes a series of demand shift variables (e.g., income, distribution, etc.—see Table 3 below for additional detail) needed for empirical analysis.

We examine three possibilities under a linear demand structure—the Choi, Raju et al. and a general linear demand system where the own price coefficients as well as the intercepts are unconstrained. As shown in the Appendix one must constrain the cross price coefficients to be equal in order for the VN to be identified and to avoid inconsistent conjectures in the MS game. The imposed constraint is:

$$A_{12} - A_{22} = 0 \tag{30}$$

The Raju et al. model holds if we have:

$$A_{11} - A_{21} = 0 \tag{31}$$

Choi, the most restrictive model, holds if in addition to 31 we have:

$$A_{10} - A_{20} = 0 \tag{32}$$

If neither 31 nor 32 hold then we have the more general linear model. Because the three demand specifications are nested, we can derive nested tests for the three specifications.

Next we develop test statistics that allow us to determine whether the data are consistent with MS or VN interaction for these three linear models. The test derived allows for the possibility that only one, both or neither form of interaction is consistent with the data. Table 1 provides the relevant test statistics. With respect to MS interaction, Equation (33a), taken from Equations 11 (Choi), 18 (Raju et al.), and A19 in the Appendix (linear) tests whether the reaction coefficient on P_2 in each equation is equal to $\gamma/2b$ with the appropriate restrictions imposed. If it is, then we conclude that the data are consistent with MS conduct. Equation (34a) in Table 1 is the analogous test statistic for the private label product.

Similarly, we test for VN interaction as follows. If Equation (33b) in Table 1 holds with the appropriate restrictions for Choi or Raju, then we conclude that VN conduct is consistent with the data for national brands. Alternatively for the general linear model, 33c must hold for VN VSI. As with MS conduct, the test statistic is the same for the Choi (Equation 11′) and Raju et al. (Equation 18′) models. Analogously, if equation (34b) with appropriate restrictions on 34c holds, then private label manufacturers are playing a VN game with retailers. In addition, an alternative test, $R_{21} = 0$, allows us to test whether the data is consistent with private label manufacturers selling to retailers at competitive prices (see Equations 15 and 19).

Table 1
Tests for VN or MS conduct

	Manufacturer Stackelberg (All Three Demand Specifications)	Vertical Nash[9] (Choi and Raju)	Vertical Nash (General Linear)
National brands	$R_{11} = \frac{A_{12}}{2A_{11}} = 0$ (33a)	$R_{11} - \frac{A_{12}}{3A_{11}} = 0$ (33b)	$R_{11} - \frac{(A_{11}A_{21} - A_{12}A_{22})A_{12}}{A_{11}(3A_{11}A_{21} - 3A_{12}{}^{2})} = 0$ (33c)
Private label	$R_{21} - \frac{A_{22}}{2A_{21}} = 0$ (34a)	$R_{21} - \frac{A_{22}}{3A_{21}} = 0$ (34b)	$R_{21} - \frac{(A_{11}A_{21} - A_{12}A_{22})A_{12}}{A_{21}(3A_{11}A_{21} - 3A_{22}{}^{2})} = 0$ (34c)

If Raju, $A_{11} - A_{12} = 0$
If Choi, $A_{11} = A_{12} = 0$, $A_{10} - A_{20} = 0$

Table 2
Proportional retail mark-up tests

	Manufacturer Stackelberg	Vertical Nash
National brands	$R_{10} - \frac{A_{10}}{2A_{11}} = 0$ (35a)	$R_{10} - \left(\frac{R_{11}A_{12}}{A_{11}}\right) = 0$ (35*b*)
Private label	$R_{20} - \frac{A_{20}}{2A_{21}} = 0$ (36a)	$R_{20}\left(\frac{R_{21}A_{22}}{A_{21}}\right) = 0$ (36b)
If Raju, $A_{11} - A_{12} = 0$		
If Choi, $A_{11} - A_{12} = 0$, $A_{10} - A_{20} = 0$		

If we determine that vertical conduct is consistent with MS or VN conduct in one or more of the three demand models, we can then test for the use of a proportional mark-up rule by retailers. Table 2 provides the relevant test statistics. They are identical for all three demand models except for the imposed restrictions. Equation (35a), for example, is based on Equations 23, 24, and A21. If the MS model holds, and Equation (35a) is satisfied, then we conclude that the category is characterized by MS vertical interaction and proportional mark-up behavior by retailers. Analogously, Equation (35b) is based on Equations 23′, 24′, and A35. If the VN model holds, and Equation (35b) is satisfied, then we conclude that the category is characterized by VN vertical interaction and proportional mark-up behavior by retailers. Equations (36a) and (36b) provide the corresponding mark-up tests for the private label product. As before, an alternative test, $R_{21} = 0$, allows us to test whether the data is consistent with private label manufacturers selling to retailers at competitive prices.

2.5. Extending the analysis to a more realistic formulation—the LA/AIDS model

In this section, we relax the linear demand assumptions of the prior models. Specifically, we specify a more flexible model of private label—national brand interaction and use non-nested tests to see if it fits the data better than the best linear specification.

Deaton and Muellbauer (1980b) originally proposed the LA/AIDS specification. Although it has been employed expensively in the economics literature, it has received scant attention in the marketing literature to date (two recent exceptions are Dreze, Nisol, and Vilcassim, 2000; Cotterill, Putsis, and Dhar, 2000). In deriving the LA/AIDS framework, Deaton and Muellbauer (1980b) begin at the level of individual utility, derive individual demand relationships and then derive the aggregation properties when applying such a model to aggregate data (e.g., the market-level data that we use here). By adding superscripts for market i (i = 1, . . . , 59) and category j (j = 1, . . . , 125), we specify the basic LA/AIDS demand model, derived by Deaton and Muellbauer (1980b) from underlying utility theory, as follows:

$$S_1^{ij} = \alpha_{10} + \alpha_{11}\, ln\, p_1^{ij} + \alpha_{12}\, ln\, p_2^{ij} + \alpha_{13}\, ln(E^{ij}/\overline{P^{ij}}) + \alpha_{14}\, ln\, D^{ij} \tag{37}$$

where:

S_1^{ij} = the dollar market share of the national brand in category i and city j,

E^{ij} = total per capita expenditure on category i in city j,
$\overline{p^{ij}}$ = Stone's price index, which is equal to $(S_1{}^{ij}\ ln\ p_1{}^{ij} + S_2{}^{ij}\ ln\ p_2{}^{ij})$, and
D^{ij} = vector of demand shift variables relevant to category i and city j.

The ratio of per capita expenditure and Stone's price index is a deflated (real) measure of per capita expenditures. Use of Stone's price index to purge expenditures of price effects gives the "linear approximate" AIDS model and allows linear estimation of the demand system (Deaton and Muellbauer, 1980a). We also assume that deflated real expenditures are exogenous and, thus, the coefficient α_{13} gives an estimate of the impact of changes in *real* (price adjusted) expenditures on demand. Contrasting with many attraction-type market share models (see, e.g., Cooper 1993), the LA/AIDS functional form is derived from the consumer's cost function and, consequently, $S_1{}^{ij}$ and $S_2{}^{ij}$ are expressed as share of expenditure. It is important to note, however, that all (quantity) demand elasticities and their respective standard errors can be recovered from the demand specification (see Green and Alston, 1990; Cotterill, Putsis and Dhar, 2000 for additional detail). Thus, any comparisons between estimated demand elasticities from the LA/AIDS model and those obtained from linear models are apples-to-apples comparisons since quantity elasticities can be directly calculated from both demand forms. Finally, we note that the private label demand equation follows by analogy.

We specifically chose the LA/AIDS model over competing alternatives for a number of reasons. First, from a theoretical perspective and as noted above, it is derived from the underlying choice axioms in utility theory. Linear (Shubik demands excepted) and logarithmic demands cannot be derived from underlying utility theory and, hence, are not necessarily consistent with the underlying choice axioms (see Deaton and Muellbauer, 1980a). Second, and particularly important for our purposes, individual behavior can be aggregated to consistently estimate demand parameters from market level data.[4] These aggregation properties make the model especially suitable for applying to market level data of the type we use here. Third, and perhaps most importantly, it gives a first-order approximation to any underlying nonlinear demand form (Deaton and Muellbauer, 1980b). Fourth, it is sufficiently flexible so as not to unduly constrain channel behavior such as price transmission and empirical estimation of market power (as per our comments in the Introduction section—see recent work by Cotterill, 1998; Cotterill, Egan, and Buckhold 2000; Tyagi, 1999). The latter two properties make the LA/AIDS form a natural point of comparison to the linear form—it can be thought of as a flexible first-order approximation to the host of nonlinear forms that may be specified as an alternative to the linear functional form. Finally, the LA/AIDS model of this type has previously been shown to describe demand and price interaction between private labels and national brands extremely well (Cotterill, Putsis, and Dhar, 2000).

We use a logarithmic first order (Taylor series) approximation to derive the following reaction functions consistent with LA/AIDS demands to produce estimable supply-side relations. Using a Taylor series expansion to obtain a linear approximate retail reaction function (consistent with the LA/AIDS specification) produces the price reaction equations specified in Equation (39) below (see Cotterill, Putsis and Dhar, 2000 for additional detail). This derivation produces four equations to be estimated: two demand equations (Equation 38) and two price reaction equations (Equation 39).

$$S_1{}^{ij} = \alpha_{10} + \alpha_{11}\, ln\, p_1{}^{ij} + \alpha_{12}\, ln\, p_2{}^{ij} + \alpha_{13}\, ln\, E^{*ij} + \alpha_{14}\, ln\, D^{ij}$$

$$S_2{}^{ij} = \alpha_{20} + \alpha_{21}\, ln\, p_1{}^{ij} + \alpha_{22}\, ln\, p_2{}^{ij} + \alpha_{23}\, ln\, E^{*ij} + \alpha_{24}\, ln\, D^{ij} \tag{38}$$

and

$$ln\, p_1{}^{ij} = \beta_{10} + \beta_{11}\, ln\, p_2{}^{ij} + \beta_{12}\, ln\, D^{ij} + \beta_{13}\, ln\, E^{*ij} + \beta_{14}\, ln\, c_1{}^{ij}$$

$$ln\, p_2{}^{ij} = \beta_{20} + \beta_{21}\, ln\, p_1{}^{ij} + \beta_{22}\, ln\, D^{ij} + \beta_{23}\, ln\, E^{*ij} + \beta_{24}\, ln\, c_2{}^{ij}, \tag{39}$$

where each of the variables are defined above and where $E^{*ij} \equiv (E^{ij}/\overline{P^{ij}})$. This general system of demand and price reaction questions derived is sufficiently general to allow MS or VN channel interaction, proportional mark-up or strategic profit maximizing behavior by retailers.

3. Empirical estimation

3.1. Data

We conduct the empirical analysis by using IRI data for six individual categories (milk, butter, bread, pasta, margarine, and instant coffee) across 59 local geographic markets for 1991 and 1992. These data were merged with independent data from *Progressive Grocer* on the demographic characteristics of the IRI geographic markets. Consistent with previous work in the private label area (e.g., Sethuraman and Mittelstaedt, 1992; Slade, 1995; Putsis and Dhar, 1998), aggregate branded and private label variables were created for each category across all markets. National brand price, feature, display, and price reduction variables are volume as opposed to dollar market share weighted averages.[5] For each model, a series of demand shift variables (D^{ij}), price equation identifying variables (c^{ij}) and a series of variables controlling for structural market characteristics (e.g., concentration) were included in the analysis. Variable definitions, based upon standard IRI measures, are provided in Table 3 below:[6]

3.2. Empirical methodology

Based on the discussion above, we estimated a series of demand and pricing models (linear and LA/AIDS, each under a variety of assumptions) via three-stage least squares (for each system, equation errors are assumed to be contemporaneously correlated across equations, but temporally uncorrelated).[7] For the linear demand model, we test for:

- Choi's (1991) linear demand model with (1) Manufacturer Stackelberg (MS) or Vertical Nash (VN) conduct and (2) strategic profit maximizing and/or proportional mark-up behavior by retailers,
- Raju, Sethuraman, and Dhar's (1995) model specifying restricted Shubik demands with (1) MS or VN conduct, and (2) strategic profit maximizing and/or proportional mark-up behavior by retailers,
- A general linear demand structure with (1) MS or VN conduct, and (2) strategic profit maximizing and/or proportional mark-up behavior by retailers.

We conducted three sets of nested tests to determine the demand structure that fit the data best. Second, to test the assumptions regarding channel behavior for each of the models studied, we also separately tested for MS and VN conduct (restrictions 33, 34) under each of the three possible demand structures. We then tested for proportional mark-up behavior on the part of retailers (restrictions 35, 36).[8]

In cases in which the more powerful nested tests proved inconclusive (e.g., we could not distinguish between MS and VN behavior in the milk category), we employed a Vuong nested test to distinguish between the competing alternatives (see Vuong, 1989; Gasmi and Vuong, 1991; and, in particular, Balasubramanian and Jain, 1994 for a discussion of the appropriateness of using the Vuong test in this situation). Based on these nested (applied first) and non-nested tests (used only in cases where we could not statistically distinguish between two alternatives by using nested tests), we were able to determine the best fitting model out of the set of possible linear models for each of the categories.

The next step was to test the best fitting linear demand specification (Equations 26 to 29) against the LA/AIDS nonlinear form (Equations 38 and 39) using a non-nested P-E test (Davidson and MacKinnon, 1981; Balasubramanian and Jain, 1994). The objective in taking this additional step was to see if we improve upon the best fitting linear model (for example, general linear demands, VN vertical strategic interaction, and retailers engaging in proportional mark-up behavior for private labels in the pasta category—see Table 4 in the next section) by incorporating flexible non-linear demands.

Table 3
Definitions for variables used in the analysis
All variables defined for the ith market, jth category)

	Dependent Variables[a]
BRSHARE	Aggregate share of category expenditure for national brands
PLSHARE	Aggregate share of category expenditure for private label products
BRPRICE	Natural log of the price of the national brand
PLPRICE	Natural log of the price of the private label product
	Demand-Shift Variables
EXPENDITURE	Natural log of per capita category expenditures deflated by Stone's price index
BRFEATURE	Percent of national brands sold with feature advertising
BRDISPLAY	Percent of national brands sold with displays and point-of-sale promotion
PLFEATURE	Percent of private label products sold with feature advertising
PLDISPLAY	Percent of private label products sold with displays and point-of-sale promotion
BRPRICEREDN	Weighted percent average price reduction, national brands
PLPRICEREDN	Weighted percent average price reduction, private label products
PLDISTN	Private label average distribution
INCOME	Natural log of the average household income in the local market
HISPANIC	Percent of population in the local market of Hispanic decent
AGE	Natural log of the average age of the local market population
	Price Equation Identifying Variables
BRVOLPUN	Natural log of average volume (weight) per package unit sold for national brand
PLVOLPUN	Natural log of average volume (weight) per package unit sold for private label
	Variables Controlling for Structural Market Characteristics
HERFINDAHL	Herfindahl index of brand concentration in the local market
GROCCR4	Percentage of all grocery sales by the top four grocery chains in the local market

[a] Price rather than the natural log of price, and per capita quantity rather than share, are used in the Choi (1991) and Raju et al. (1995) models.

Finally, we note that each of the systems estimated necessitates deriving and estimating a relatively complicated set of demand and price equations. We do so because estimating a *system* of equations where price reactions and demand are endogenously and simultaneously determined are crucially important for obtaining unbiased parameter estimates. Two recent articles (Bayus and Putsis, 1999; Cotterill, Putsis, and Dhar, 2000) strongly suggest that ignoring the endogeneity and simultaneity of price determination can lead to not only biased, but also wrong parameter estimates. This combined with the fact that we are attempting to empirically assess both demand (functional form assumptions) and supply (within channel interaction and retailer behavior) side assumptions necessitates taking the systems approach that we do.

4. Empirical results

Table 4 summarizes the results from the hypothesis tests used to determine (1) whether the Choi (1991), Raju et al. (1995), or the more general linear demand model best fit the data, (2) which form of vertical strategic interaction best fits the data, and (3) whether proportional mark-up behavior by retailers is consistent with the data.

Examining Table 4, we find that the Choi (1991) demand model fits best for the bread category, while the Raju et al. (1995) modified Shubik demands are more consistent with the milk and butter categories. Three products, pasta, margarine and instant coffee require the

Table 4
Demand form, vertical strategic interaction (MS or VN), and proportional mark-up test results[a] for individual categories, linear models[b]

Category	National Brand or Private Label	Demand Form	Vertical Strategic Interaction	Retailer Behavior
Milk	National Brand	Raju	Vertical Nash[cd]	PM[c]
	Private Label	Raju	Vertical Nash[cd]	Not PM[c]
Butter	National Brand	Raju	Vertical Nash[cd]	Not PM[cd]
	Private Label	Raju	Vertical Nash[c]	Not PM[cd]
Bread	National Brand	Choi	Vertical Nash[cd]	Not PM[cd]
	National Brand	Choi	Vertical Nash[cd]	Not PM[cd]
Pasta	National Brand	Linear	Manufacturer Stackelberg[cd]	Not PM[d]
	Private Label	Linear	Vertical Nash[d]	PM[c]
Margarine	National Brand	Linear	Vertical Nash[c]	Not PM[cd]
	Private Label	Linear	Vertical Nash[d]	Not PM[c]
Instant Coffee	National Brand	Linear	Manufacturer Stackelberg[c]	Not PM[d]
	Private Label	Linear	Vertical Nash[cd]	Not PM[d]

[a]Unless otherwise noted, all tests employ a nested likelihood ratio (LR) test at $\alpha = .05$ based on the test statistics summarized in Table 1 and Table 2.

[b]As noted in the text, the three linear models considered are the general linear model (denoted Linear above) and the general linear model with the Choi and Raju et al. restrictions imposed (denoted Choi and Raju, respectively).

[c]Denotes that the result is significant at $\alpha = .05$ or better.

[d]Denotes the result is based upon the use of a Vuong non-nested test as discussed in the text.

PM Denotes proportional mark-up behavior by retailers.

Not PM Denotes that the tests do not support proportional mark-up behavior by retailers.

more general linear demand specification where own price coefficients are not constrained to identical values.

Out of 12 tests for VSI (6 for national brands and 6 for private labels), 10 suggested VN interaction (8 of these were significant at $\alpha = .05$) and only two suggested MS vertical conduct (only 1 significant at $\alpha = .05$). Perhaps not surprisingly, all 6 private label products supported VN vertical conduct (all but 1 significant). The vertical interaction for national brands was a bit more idiosyncratic to the category—pasta and instant coffee seem to be consistent with MS conduct (although the coffee result is not significant) while the other four categories exhibit VN interaction. Such variability in vertical strategic interaction is consistent with the variation observed by Putsis and Dhar (1998) with respect to horizontal strategic interaction. We note, however, that Sudhir (2000) observed MS conduct (not VN) for two categories of national brands. If we had limited our study to national brands in the pasta and instant coffee categories, we may have come to similar conclusions. Our results highlight the importance of performing analyses over multiple categories. Further, our results suggest that simply *assuming* MS conduct in the channel—particularly for private labels—may be inappropriate.

We find similarly disconcerting results for empirical studies that employ proportional mark-up assumptions. Out of 12 tests for proportional mark-up behavior, only 2 (national brand milk and private label pasta) support the use of proportional mark-ups by retailers. Our finding that retailers do not appear to employ proportional mark-ups generally is consistent with the findings of Sudhir (2000) for the yogurt and peanut butter categories.

What can we say about the nonlinear demand structure? We present two sets of results. Table 5 presents the estimated demand and price reaction elasticities for the LA/AIDS functional form. These parameter estimates have not only a great deal of face validity, but are also consistent with previous research on a number of dimensions. For example, Tellis (1988) in a meta-analysis of reported demand elasticities, found the mean price elasticity of demand to be -1.71, consistent with the national brand elasticities reported in the first row of Table 5. More generally, the magnitudes of the own and cross price elasticities by category are also consistent with those reported in recent research by Danaher and Brodie (1998). In terms of price reaction elasticities (for a national brand price change, e.g., this is measured as the percent change in the national brand price relative to the percent change in private label price), the price reactions of national brands were small in magnitude, with the highest price reaction by national brands occurring in the category with one of the highest private label shares (margarine), consistent with expectations.

Overall, the reported price reactions are very close to those reported by Lambin (1976) and others (e.g., Hanssens, Parsons and Schultz, 1990, pp. 201–210, report a series of price elasticities from other studies under a variety of settings; see the left-hand column in Table 6–9, page 206, in particular).

In contrast, the results for the Choi (1991) and Raju et al. (1995) linear demand specifications (Table 6) show fewer significant coefficients and a great deal more volatility in the parameter estimates. While there is some consistency between the magnitude of the *significant* demand elasticities for the linear and the LA/AIDS specifications, there are not many significant parameter estimates in Table 6. For both the linear demand and price reaction elasticities, a number of parameter estimates are outside the range reported in other studies.

Table 5
Demand and reaction elasticities for individual product categories, LA/AIDS model

	Milk	Butter	Bread	Pasta	Margarine	Inst. Coffee
BR own price elasticity	−1.63 (−4.20)[a]	−1.82 (−3.19)[a]	−1.80 (−9.66)[a]	−1.48 (−13.90)[a]	−1.16 (−21.63)[a]	−1.05 (−47.06)[a]
PL own price elasticity	−1.22 (−2.81)[a]	−2.93 (−4.47)[a]	−1.66 (−4.66)[a]	−2.31 (−3.38)[a]	−5.86 (−6.60)[a]	−0.100 (−0.318)
BR cross price elasticity	0.458 (0.513)	1.39 (2.94)[a]	0.234 (1.86)	0.206 (1.91)	0.474 (5.47)[a]	−0.043 (−2.86)[a]
PL cross price elasticity	0.308 (1.63)	1.14 (1.44)	22.6 (4.30)[a]	3.04 (4.50)[a]	1.66 (3.01)[a]	1.05 (2.26)[b]
BR price reaction elasticity	−0.600 (−1.64)	0.401 (3.40)[a]	0.322 (3.39)[a]	1.25 (0.932)	1.06 (3.69)[a]	0.070 (1.07)
PL price reaction elasticity	0.175 (2.10)[b]	0.777 (4.53)[a]	0.321 (0.84)	1.01 (2.89)[a]	0.231 (0.776)	−0.083 (−0.518)
Average BR share	.30	.54	.62	.81	.84	.94
NOBS	116	112	118	118	118	108

BR = National Brand, PL = Private Label; t-statistics in parentheses.
[a] Significant at the 1% level;
[b] Significant at the 5% level.

Table 6
Demand and reaction elasticities for individual product categories, linear models

	Milk	Butter	Bread	Pasta	Margarine	Inst. Coffee
Selected demand structure	Raju	Raju	Choi	Linear	Linear	Linear
BR own price elasticity	−2.672 (−6.03)[a]	−0.189 (−0.26)	−0.574 (−1.72)	−1.203 (−9.04)[a]	−1.281 (−13.45)[a]	−0.942 (−20.12)[a]
PL own price elasticity	−1.448 (−4.28)[a]	−0.887 (−0.96)	−0.042 (−0.06)	−0.268 (−0.46)	−7.51 (−5.50)[a]	−0.052 (−0.15)
BR share cross price elasticity	1.500 (3.13)[a]	−0.572 (−0.88)	−0.445 (−1.68)	−0.034 (−0.39)	0.443 (4.43)[a]	−0.003 (−0.14)
PL share cross price elasticity	0.691 (3.13)[a]	−0.797 (−0.88)	−1.253 (−1.68)	−0.223 (−0.39)	4.470 (4.43)[a]	−0.060 (−0.14)
BR price reaction elasticity	−0.298 (1.19)	0.544 (5.00)[a]	0.293 (2.09)[b]	1.358 (0.457)	1.323 (4.37)[a]	0.143 (2.27)[b]
PL price reaction elasticity	0.217 (2.77)[a]	0.782 (5.73)[a]	0.610 (2.14)[b]	2.061 (3.72)[a]	0.260 (0.87)	0.077 (0.05)
Average BR share	.30	.54	.62	.81	.84	.94
NOBS	116	112	118	118	118	108

BR = National Brand, PL = Private Label; t-statistics in parentheses.
[a] Significant at the 1% level;
[b] Significant at the 5% level.

For example, the estimated price reaction elasticity of 2.1 (significant at $\alpha = .01$) for private labels in the pasta category not only *seems* high, it is inconsistent with previous research (e.g., Lambin 1976, see Hanssens, Parsons, and Schultz, 1990, p. 206). We suggest that the relative stability of the parameter estimates and the high number of significant coefficients in the LA/AIDS system are due in large part to the flexible form of the LA/AIDS specification. Note that in Table 6, four of the six price private label reaction elasticities are significantly different from zero. For these four products this is sufficient evidence to reject the hypothesis that private label suppliers are price takers.

Finally, we tested the two demand structures more formally. Specifically, we tested for the possibility of a simpler linear functional form by comparing the LA/AIDS specification to the linear form using a non-nested P-E test (Davidson and MacKinnon, 1981). Balasubramanian and Jain (1994) suggest that the choice of non-nested test should be guided by the circumstances surrounding the test (see, e.g., their Table 7 for the appropriateness of using the P-E test in the current application). Jain and Vilcassim (1989) demonstrate that the sample size requirements for the P-E test may be less stringent than that required for Lagrange multiplier tests, suggesting that that it is particularly relevant in this particular application. We employ it as detailed in Greene (1997, pp. 459–462). The results for the demand specification were even more conclusive than those for within-channel structure discussed above. For all six categories, the P-E test cleanly rejected the null of a linear model at $p \ll .01$.

5. Conclusions, limitations, and future research

In the introduction of this research, we set forth our objective to examine three central assumptions common to much of the existing channels literature. Using data spanning 2 years for six different product categories, we reached clear conclusions on each:

> *Form of Vertical Strategic Interaction between Manufacturers and Retailers*. Although the empirical results pertaining to VSI generally support the VN model of channel interaction, the results vary by category and by type of brand (national brand versus private label). VN interaction seems to be standard for private labels, for example, whereas vertical strategic interaction for national brand vary from VN to MS depending on the category. As noted above, this finding of cross-category and cross-brand variability in *vertical* strategic interaction is consistent with recent research by Putsis and Dhar (1998) addressing *horizontal* strategic interaction (HSI). This suggests researchers cannot arbitrarily assume a form of vertical interaction and move on with their analyses. Since vertical strategic interaction between manufacturers and retailers appears to be idiosyncratic to the category and brand, researchers must now consider multiple forms of vertical interaction in order to produce more general results. Their conclusions must also be evaluated for their sensitivity to their assumptions on vertical interaction.
>
> *The Use of Nonstrategic Proportional Mark-up Behavior by Retailers*. We generally reject the use of proportional mark-up behavior by retailers, consistent with recent research by Sudhir (2000). These results suggest that models specifying proportional mark-up behavior do not accurately reflect market reality. Combined with the identification problem inherent in some empirical applications, our results indicate that researchers must now address strategic price interaction between retailer and manufacturer explicitly.

Linear Demands. We reject linear demands in favor of more flexible forms, such as the LA/AIDS model used here. However, it is instructive to put this in perspective. Despite some restrictive assumptions and a seemingly inflexible functional form, we were able to determine which of the two restricted linear demand models (Choi, 1991; Raju et al., 1995) best fit the data and derive a number of statistically significant and reasonable elasticities. However, the volatility of the parameter estimates is disconcerting. The large number of insignificant coefficients leaves a researcher without information on key parameter values. Further, as demonstrated elsewhere, linear demands place rather restrictive assumptions on channel behavior. For example, assuming a linear demand implies a fixed and constant pass-through rate within the channel, imposing a rather restrictive constraint on vertical strategic behavior. In contrast, the LA/AIDS framework, discussed in detail above, provides us with a flexible functional form that performs well on a number of individual categories and does not require similarly restrictive assumptions on vertical behavior. Non-nested tests suggested that the improvement in fit in moving to the nonlinear form was significant. All of this suggests that demand functional form matters—hence future theoretical and empirical research addressing channel issues should avoid the linear form wherever possible.

This last point illustrates the value of empirical analysis of the type conducted here. Empirical analysis should be viewed as a complement to theoretical research, not as a challenge to it. Empirical research can guide which aspects of the theory warrant further investigation, and as such it should not be limited to testing the implications of the theory. For example, we find that linear demands do not fit nearly as well as nonlinear forms, result in questionable demand-side parameter estimates, and place rather restrictive assumptions on both demand and channel behavior. Consequently, future research in the channels area—empirical as well as theoretical—should be cautious in using linear demands.

Nevertheless, readers should be cautioned about a number of limitations of our research. For example, additional product categories and models need to be examined. We studied only six categories. A more extensive examination across multiple categories, and a deeper understanding of the cross-category variation in observed channel interaction, is also needed. As of now, we have little guidance as to what structures to expect under which circumstances. In addition, we examine a very limited set of models and demand structures. This was intentional with the objective of keeping the analysis focused and rigorous. However, this clearly limits our ability to generalize our findings to other assumptions of channel behavior, other models of channel relationships and alternative demand functional forms. Finally, given the restrictions placed on vertical conduct as a result of a linear demand specification, a comprehensive empirical investigation of nonlinear forms, channel behavior and related pass through rates is needed (see Cotterill, 1998; Cotterill et al., 2000; Tyagi, 1999 for related theoretical relationships).

Nonetheless, the results above send several clear messages for theoretical research in channels. First, models addressing vertical interaction must consider multiple forms of VSI, examining the robustness of the findings to VSI assumptions. Second, researchers should avoid the proportional mark-up assumption and linear demands. The most immediate need for future research is to identify the form of VSI likely to exist in specific circumstances so that we have a better sense of when to specify VN versus MS interaction.

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Appendix: Derivation of the general linear model reaction functions and nested test statistics

In the main body of this article we analyze two restricted versions of the general linear demand model: the Choi and the Raju et al. (restricted Shubik) demand models. In this appendix we analyze the unrestricted linear model. We derive the corresponding price reaction and test statistics for Manufacturer Stackelberg (MS), Vertical Nash (VN), and markup behavior by retailers. We also will show that a totally unconstrained general linear model is not internally consistent (MS case) and not identified (VN case).[1] To obtain internal consistency and to identify and estimate linear models one must impose a cross price equality constraint ($\gamma_1 = \gamma_2$ in A1 and A2). This observation is an important one. Once we account for vertical relationships, naively estimating an unconstrained linear demand model is inappropriate. This has important implications for all empirical work using linear demands and it highlights one additional concern when employing a linear demand specification. To illustrate, we note that the general linear demand model for two goods is:

$$q_1 = a_1 + b_1 p_1 + \gamma_1 p_2 \tag{A1}$$

$$q_2 = a_2 + b_2 p_2 + \gamma_2 p_1 \tag{A2}$$

The retailer maximizes category profits, π_R by choosing values of p_1 and p_2 for given wholesale prices for the product, w_1, w_2:

$$\underset{p_1 p_2}{\text{MAX}}\ \Pi_R = (p_1 - w_1)q_1(p_1, p_2) + (p_2 - w_2)q_2(p_1, p_2) \tag{A3}$$

The two manufacturers maximize their profits as follows (c_1 and c_2 denote the respective manufacturer marginal costs):

$$\underset{w_i}{\text{MAX}}\ \Pi_i = (w_i - c_i)q_i(p_1, p_2) \qquad i = 1, 2 \tag{A4}$$

[1] Lack of consistent conjectures is a problem in many popular oligopoly models including the Cournot model. While unattractive, it has not precluded the use of such models in research. However, the identification problem is an econometrically important issue because it prevents empirical analysis of the model.

The retailer's first order conditions (FOC) are:

$$-2b_1p_1 + (\gamma_1 + \gamma_2)p_2 + b_1w_1 - \gamma_2w_2 + a_1 = 0 \tag{A5}$$

$$-2b_2p_2 + (\gamma_1 + \gamma_2)p_1 + b_2w_2 - \gamma_1w_1 + a_2 = 0 \tag{A6}$$

Solving for the Manufacturer Stackelberg Reaction Function Assuming Retail Profit Maximization

As in the text, the procedure for obtaining MS reaction functions between p_1 and p_2, starts with solving A5 and A6 for the retail reaction functions. In these, p_1 will be a function of w_1, w_2 and parameters; and, p_2 will be a function of the same. We can solve former for w_1 and a function of p_1, p_2 and parameters. This equation is substituted into A4 for w_1 and manufacturer one maximizes profits by choosing p_1. The corresponding first order condition given below will then be solved for the manufacturer's MS reaction function in retail prices. We start by solving A5 and A6 for the retailer's reaction functions, which are:

$$p_1 = \frac{G_1w_1}{D} + \frac{N_1w_2}{D} + \frac{A_1}{D} \tag{A7}$$

$$p_2 = \frac{G_2w_2}{D} + \frac{N_2w_1}{D} + \frac{A_2}{D} \tag{A8}$$

where:

$$\begin{aligned}
D &= 4b_1b_2 - (\gamma_1 + \gamma_2)^2 \\
G_2 &= 2b_1b_2 - \gamma_2(\gamma_1 + \gamma_2) \\
N_2 &= b_1(\gamma_1 + \gamma_2) - 2\gamma_1b_1 \\
A_1 &= 2b_2a_1 + (\gamma_1 + \gamma_2)a_2 \\
A_2 &= 2b_1a_2 + (\gamma_1 + \gamma_2)a_1
\end{aligned}$$

Next we solve A7 and A8 for w_1 and w_2:

$$w_1 = \frac{DG_2}{G_1G_2 - N_1N_2}p_1 - \frac{N_1D}{G_1G_2 - N_1N_2}p_2 - \frac{G_2A_1}{G_1G_2 - N_1N_2} + \frac{N_1A_2}{G_1G_2 - N_1N_2} \tag{A9}$$

$$w_2 = \frac{DG_1}{G_1G_2 - N_1N_2}p_2 - \frac{N_2D}{G_1G_2 - N_1N_2}p_1 - \frac{G_1A_2}{G_1G_2 - N_1N_2} + \frac{N_2A_1}{G_1G_2 - N_1N_2} \tag{A10}$$

We now substitute A9 into manufacturer one's profit maximization problem (A4) since manufacturer one knows the retailer's reaction function, she can maximize profits by choosing p_1 rather than w_1. Note that this assumes that when manufacturer one changes p_1 in A9 that p_2 does not change. This is not a consistent conjecture because we will show below that the reaction coefficient is not zero. If we assume $\gamma_1 = \gamma_2$ in the demand model, then N_1 and N_2 are identically zero (and p_2 drops out of A9). Also, p_1 drops out of A10, so the contradiction disappears. Note also that w_2 drops out of A7 and w_1 drops out of A8 so retailers react only to changes in a product's own wholesale price.

In our derivation, however, we will not assume $\gamma_1 = \gamma_2$ so that we can show the general solution. The MS reaction function for manufacturer one is:

$$p_1 = \left(\frac{b_1 N_1 + \gamma_1 G_2}{2b_1 G_2}\right) p_2 + \frac{a_1 DG_2 + b_1 G_2 A_1}{2b_1 + DG_2} + \frac{b_1(G_1 G_2 - N_1 N_2)}{2b_1 DG_2}\, c_1 \tag{A11}$$

This is the structural form of the estimating equation (28) in the text. Expanding the reaction coefficient into parameters of the original demand model (A1, A2) gives:

$$R_{11} = \frac{b_1 N_1 + \gamma_1 G_2}{2b_1 G_2} = \frac{b_1 b_2(\gamma_1 + \gamma_2) - 2b_1 b_2 \gamma_2 + 2b_1 b_2 \gamma_1 - \gamma_1 \gamma_2(\gamma_1 + \gamma_2)}{4{b_1}^2 b_2 - 2b_1 \gamma_2(\gamma_1 + \gamma_2)} \tag{A12}$$

Now if $\gamma_1 = \gamma_2$, one obtains:

$$R_{11} = \frac{2\gamma(b_1 b_2 - \gamma^2)}{4b_1(b_1 b_2 - \gamma^2)} = \frac{\gamma}{2b_1} \tag{A13}$$

Finally, if we constrain $b_1 = b_2$ we obtain the Choi and Raju et al. reaction coefficient derived in the text. By symmetry the reaction coefficient for manufacturer two when $\gamma_1 = \gamma_2$ is:

$$R_{21} = \frac{\gamma}{2b_2} \tag{A14}$$

The test statistics for MS in this symmetric cross price linear model using the nomenclature of the text (Equations 26–29) are:

$$R_{11} - \frac{A_{12}}{2A_{11}} = 0 \tag{A19}$$

$$R_{21} - \frac{A_{22}}{2A_{21}} = 0 \tag{A20}$$

where: A_{10} is not constrained to equal A_{20}, and A_{11} is not constrained to equal A_{21}.

Solving for MS Reaction Functions Assuming Retail Proportional Retail Markups

If the retailer uses a proportional mark-up strategy, then, by definition, the following equation replace the first order conditions A5 and A6:

$$P_1 = m_1 w_1 \tag{A15}$$

$$P_2 = m_2 w_2 \tag{A16}$$

Solving for w_1 and w_2 gives:

$$w_1 = k_1 p_1 \tag{A15$'$}$$

$$w_2 = k_2 p_2 \tag{A16$'$}$$

where:

$$k_1 = \frac{1}{m_1} \text{ and } k_2 = \frac{1}{m_2}.$$

Substituting A15′ into A4 for manufacturer one and maximizing profits by choosing p_1 gives the following reaction function:

$$p_1 = \frac{\gamma_1}{2b_1} p_2 + \frac{a_1}{2b_1} + \frac{c_1}{2k_1} \tag{A17}$$

By symmetry the reaction function for the second manufacturer is:

$$p_2 = \frac{\gamma_2}{2b_2} p_1 + \frac{a_2}{2b_2} + \frac{c_2}{2k_2} \tag{A18}$$

If we impose cross symmetry to avoid inconsistent conjectures, i.e., $\gamma_1 = \gamma_2$, the reaction coefficients under proportional markups are identical to those under profit maximization in the MS case. The test for markup conduct, given MS, again as in the text involves the intercept term in the estimated reaction equations. The test statistics are:

$$R_{10} - \frac{A_{10}}{2A_{11}} = 0 \tag{A21}$$

$$R_{20} - \frac{A_{20}}{2A_{21}} = 0 \tag{A22}$$

where A_{10} is not constrained to equal A_{20} and A_{11} is not constrained to equal A_{21}.

Solving for VN Reaction Functions Assuming Retail Profit Maximization

To solve the VN problem we need each manufacturers first order condition for (A4) as well as the retailers FOC expressed as (A7) and (A8). The two manufacturer FOCs are:

$$-b_1p_1 + \gamma_1p_2 - b_1w_1 + b_1c_1 + a_1 = 0 \tag{A23}$$

$$-b_2p_2 + \gamma_2p_1 - b_2w_2 + b_2c_2 + a_2 = 0 \tag{A24}$$

Solving these for w_1 and w_2 gives:

$$w_1 = -p_1 + \frac{\gamma_1}{b_1} p_2 + c_1 + \frac{a_1}{b_1} \tag{A25}$$

$$w_2 = -p_2 + \frac{\gamma_2}{b_2} p_2 + c_2 + \frac{a_2}{b_2} \tag{A26}$$

Substituting A25 and A26 into A7 gives manufacturer one's reaction function:

$$p_1 = \left[\left(\frac{G_1\gamma_1}{Db_1} - \frac{N_1}{D}\right)/F\right]p_2 + \frac{G_1c_1}{DF} + \frac{N_1c_2}{DF} + \frac{G_1}{DF}\frac{a_1}{b_1} + \frac{N_1}{DF}\frac{a_2}{b_2} + \frac{A_1}{DF} \tag{A27}$$

where:

$$F = 1 + \frac{G_1}{D} - \frac{N_1}{D}\frac{\gamma_1}{b_2}$$

Again, as in the MS case, this is a very complicated function. Note also that c_2 as well as c_1 appears in it so this function is not identified for estimation purposes. However, if we impose $\gamma_1 = \gamma_2$ in the demand equations $N_1 = 0$, the c_2 term vanishes and the equation is identified. Also the reaction coefficient reduces to:

$$R_{11} = \frac{(b_1b_2 - \gamma^2)\gamma}{b_1(3b_1b_2 - 3\gamma^2)} \tag{A28}$$

By symmetry, for manufacturer 2:

$$R_{21} + \frac{(b_1b_2 - \gamma^2)\gamma}{b_2(3b_1b_2 - 3\gamma^2)} \tag{A29}$$

The corresponding test statistics using the nomenclature of Equations 26–29 in the text are:

$$R_{11} - \frac{(A_{11}A_{21} - A_{12}A_{22})A_{12}}{A_{11}(3A_{11}A_{21} - 3A_{12}{}^2)} = 0 \tag{A30}$$

$$R_{21} - \frac{(A_{11}A_{21} - A_{12}A_{22})A_{12}}{A_{21}(3A_{11}A_{21} - 3A_{22}{}^2)} = 0 \tag{A31}$$

where A_{12} is constrained to equal A_{22}. Finally note that if $b_1 = b_2$ A28 and A29 reduce, as they should, to the coefficients for the Raju et al. and Choi models derived in the text:

$$R_{11} = R_{21} = \frac{\gamma}{3b} \tag{A32}$$

Solving for VN Reaction Functions Assuming Retail Proportional Markup

If vertical conduct is Nash then we solve A15 and A16 the markup equations with the two manufacturer FOC A23 and A24 to obtain following reaction functions:

$$p_1 = \left(\frac{m_1}{1 + m_1}\right)\frac{\gamma_1}{b_1}p_2 + \frac{a_1}{b_1}\left(\frac{m_1}{1 + m_1}\right) + c_1\left(\frac{m_1}{1 + m_7}\right) \tag{A33}$$

$$p_2 = \left(\frac{m_2}{1 + m_2}\right)\frac{\gamma_2}{b_2}p_1 + \frac{a_2}{b_2}\left(\frac{m_2}{1 + m_2}\right) + c_2\left(\frac{m_2}{1 + m_2}\right) \tag{A34}$$

The corresponding tests are the same as those in the Choi and Raju et al. demand structure:

$$\frac{R_{10}}{R_{11}} = \frac{A_{12}}{A_{10}} \tag{A35}$$

$$\frac{R_{20}}{R_{21}} = \frac{A_{22}}{A_{20}} \tag{A36}$$

However, note that here, the intercepts and own price coefficient are not constrained to be equal, an important distinction. Thus, what we reference as the "general linear demand" in the text is indeed considerably more general than the Choi and Raju et al. restricted Shubik demand models.

Notes

1. More precisely, under VN interaction, each manufacturer chooses its wholesale price conditional on both the retailer's margin on its own product and the observed retail price of the competing brand, whereas the retailer determines the margin of each brand conditional on the wholesale prices. Under MS interaction, each manufacturer chooses its wholesale price based on the retailer's best response function, conditional on the wholesale price of its competitor; the retailer determines the price of each product so as to maximize the total profit from both brands given the manufacturer's wholesale prices. Thus, each form of vertical interaction captures a very different form of competitive interaction across players within the channel. For additional detail, see Choi (1991).
2. We simplified presentation of the restricted Shubik demands here to facilitate comparison with the Choi (1991) specification. Under the Raju et al. restricted Shubik demands, the national brand demands are expressed as follows: $q_1 = 1/(1 + \alpha)[1 - p_1 + \delta(p_2 - p_1)] = [1/(1 + \alpha)] - [(1 + \delta)/(1 + \alpha)]p_1 + [\delta/(1 + \alpha)]p_2$, which is simplified to Equation 16 above. For the private label, the only difference is that the intercept, a_2 above, is equal to $[a/(1 + a)]$, as opposed to $[1/(1 + \alpha)]$ for the national brand. The (accordingly) appropriate demand restrictions are incorporated in estimation.
3. As pointed out by Genesove and Mullin (1998), the requirement that $k_1 = 2$ is a direct consequence a linear demand schedule, which implies that exactly 50% of any change in cost (c_1 to the manufacturer and w_1 for the retailer) is passed on. Thus, because of double marginalization, only 1/4 of a change in c_1 (see Equation 11) and c_2 (Equation 14) is eventually passed on to the consumer. As pointed out by Genesove and Mullin (1998) and by Cotterill, Putsis and Dhar (2000), what seems like a simple demand assumption (linearity), actually places a rather restrictive assumption (non-strategic pass through of 50%) on channel pricing behavior. See also recent work by Lee and Staelin (1997) and Tyagi (1999) for relevant discussions.
4. The LA/AIDS aggregation properties are especially important for our purposes. First, note that the LA/AIDS is PIGLOG (*P*rice *I*ndependent *G*eneralized *LOG*arithmic) in form, which does not require the assumption of parallel linear Engel curves. This implies that we are able to consistently estimate expenditure effects using linearly aggregated data. In addition, under the assumption that prices change proportionately from period to period

across retailers, the LA/AIDS demand equation eliminates any linear response aggregation bias. See Cotterill, Putsis, and Dhar (2000) for additional detail.

5. For example, aggregate private label and national brand variables were created for share, price and price reduction. Private label (national brand) share is sum of all private label (national) brands in the ith market, jth category. Private label (national brand) price is the volume-weighted average price of all private labels (national brands) in the ith market, jth category. The two price reduction variables are volume-weighted percent price reduction for all private label and branded products, respectively. Thus, for price and share, we have four aggregate variables: total branded share, total private label share, volume-weighted average price of national brands, and the volume-weighted average price of private label products. Also, note that the choice of variables was influenced by data availability. For example, no coupon information was available, whereas average age, income, and percent Hispanic were the only local demographic variables available.
6. To be consistent with previous research employing scanner data, standard IRI measures were used wherever appropriate. IRI relies on in-store visits and individual store-level scanner data to compile the measures used here. For additional detail, see the *IRI Marketing Factbook*, which is published annually. All volume measures/weights used are ACV (All Commodity Volume) measures as defined by IRI.
7. Although time series approaches to estimate reaction functions are more common, cross-sectional approaches, such as the one used here, are not at all uncommon. For a succinct rationale for using a cross-sectional approach in such studies, see Baker (1998).
8. An alternative specification (as suggested by Raju, et al. 1995) would be to examine the price difference (or "gap") between brands. However, our profit maximization (and mark-up) specification is more general than the gap model because it specifies each price (p1, p2, w1, w2) as a strategic variable rather than just the price gap (e.g., p1 − p2). However, it might be useful in future research to test for the appropriateness of the gap model.
9. See Appendix for derivation of these test statistics.

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